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An analysis of heat transfer using equivalent thermal conductivity of liquid phase during melting inside an isothermally heated horizontal cylinder

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Abstract—A method of analysis using the equivalent thermal conductivity of the liquid phase is presented for the melting process of a phase change material (PCM) inside an isothermally heated horizontal cylinder. The equivalent thermal conductivity of the liquid phase during the melting process is estimated from the results for natural convection heat transfer between concentric horizontal circular cylinders. The melting process is, thus, analyzed as a pure heat conduction problem. Experiments are also performed using n-octadecane as a PCM. The present analytical results for melting rates show good agreement with the experimental data for a wide range of Rayleigh numbers. It is also shown that the effect of natural convection on the melting process can be negligible for $Ra_D < 10^5$.

INTRODUCTION

MELTING or solidification of a phase change material (PCM), which introduces heat transfer of latent heat accompanied with phase change, is of great interest in engineering applications. In recent years, the latent heat of phase change has become widely utilized, for example, for the effective utilization of solar energy and for the latent heat storage of a heat-pump air conditioning system.

For the melting and solidification processes inside a horizontal cylinder, a number of researches have been reported for the heat transfer as well as phase change phenomena. For the solidification process, which can be treated as a pure heat conduction problem, analytical as well as experimental studies have been performed. On the other hand for the melting process, which involves the natural convection effect of the liquid phase, numerical calculations [1–3] have been applied instead of an analytical method. It seems that the heat transfer characteristics for both the melting and solidification processes inside a horizontal cylinder have been revealed. In engineering applications, however, a latent heat storage system consists of a bundle of PCM capsules. Therefore, a study for the heat transfer characteristics of a bundle of PCM capsules is practically important.

An analysis on the heat transfer characteristics in the solidification process for the fifty-row latent heat storage capsules located in convective flow was made [4]. It was revealed that there is a certain location in the rows where the heat transfer rate between the capsules and the convective fluid shows a maximum value, and that the location moves to the downstream rows as solidification proceeds. For the melting process, the occurrence of natural convection in the liquid phase makes it difficult to analyze the heat transfer characteristics. As far as a single horizontal cylinder is concerned, some numerical calculations have been reported for the melting process as described above. It is, however, not convenient to apply the numerical method to all of the bundle of capsules.

In the present study, a method of analysis using an equivalent thermal conductivity of the liquid phase is presented for the melting process of PCM inside an isothermally heated horizontal cylinder. The analytical results are compared with the experimental data obtained by using n-octadecane as a PCM. In this study, the case in which the unmelted solid is fixed at its position during melting is considered.

EXPERIMENTS

Test apparatus

The experimental apparatus consists of a test cylinder, two constant-temperature water baths and water circulation systems. The test cylinder consists of a 257 mm long copper tube, 50 mm i.d., having a wall thickness of 2.0 mm as shown in Fig. 1. The copper cylinder, filled with n-octadecane [$CH_3(CH_2)_{16}CH_3$, 99% pure], was closed at both ends by transparent acrylic resin plates so that the melting process could be observed. Heating of the cylinder was carried out by circulating hot water from the constant temperature bath through the water chambers outside the cylinder. Three water chambers were provided to obtain a constant and uniform wall temperature. The cylinder wall temperature, T_w, measured at three locations along the cylinder axis, were ascertained to coincide with each other. The volumetric expansion associated with phase change from solid to liquid is accommodated by an overflow outlet

NOMENCLATURE

$A(\tau)$	dimensionless area of unmelted solid, $d^2(\tau)/D^2$	t	time
$d(\tau)$	equivalent diameter of unmelted solid	T_f, T_w	melting temperature of PCM and cylinder wall temperature.
D	inner diameter of cylinder		
$k_{eq}(t)$	equation (4)	Greek symbols	
L	gap width of concentric cylinders	β	volume expansion coefficient
L_f	latent heat of fusion	ε	eccentricity, equation (1)
Nu_d, Nu_D	Nusselt numbers, equations (3)	κ	thermal diffusivity
Pr	Prandtl number, ν/κ	$\lambda_{eq}(t)$	equivalent thermal conductivity of liquid, equation (4)
$q_L(t)$	rate of heat transferred from cylinder to solid–liquid interface	λ_l	thermal conductivity of liquid
$q_L^*(\tau)$	dimensionless value of $q_L(t)$	ν	kinematic viscosity
Ra_d	Rayleigh number, $g\beta(T_w-T_f)d^3/\nu\kappa$	ρ_l, ρ_s	densities of liquid and solid
Ra_D	Rayleigh number, $g\beta(T_w-T_f)D^3/\nu\kappa$	τ	dimensionless time, $\lambda_l(T_w-T_f)t/[\rho_l L_f(D/2)^2]$.
Ra_L	Rayleigh number, $g\beta(T_w-T_f)L^3/\nu\kappa$		

at one end of the cylinder. To avoid sinking of the unmelted solid to the bottom of the cylinder, eight wooden sticks were located on each acrylic resin plate at both ends.

Test procedure

Before the experiments, water the temperature of which was controlled slightly lower than the melting temperature of the PCM, was circulated through the water chambers. After steady-state conditions of the PCM in the cylinder were reached, the temperature controlled hot water was circulated through the chambers and, thus, the melting experiments were started. In these experiments, the sensible heat which was necessary to raise the initial solid PCM temperature to the melting temperature was about 2% of the total latent heat of the PCM and was, therefore, neglected. During the melting experiments, the solid–liquid interface was photographed at predetermined time intervals to measure the melting rate. The melted and unmelted regions of the photographs were cut out, respectively, and the melting rate was evaluated by measuring the weight of both the melted and unmelted regions with a precision balance. The accuracy of this device was estimated to be about $\pm 2\%$ for an area of 19.6 cm^2. Two dimensionality of the heat flow from the cylinder wall to the solid–liquid interface was ascertained by the fact that the shape of the melting PCM showed almost two dimensionality. The equivalent diameter of the unmelted solid, $d(\tau)$, was estimated from the diameter of the circle the area of which coincided with that of the unmelted solid. The melting temperature of n-octadecane was estimated from the experiments as $T_f = 27.8°C$. The experimental ranges covered in the present experiments were $30.1°C \leqslant T_w \leqslant 47.8°C$ and $9.62 \times 10^6 \leqslant Ra_D \leqslant 7.97 \times 10^7$.

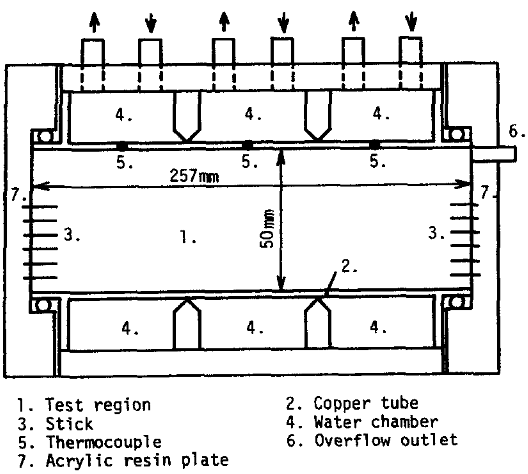
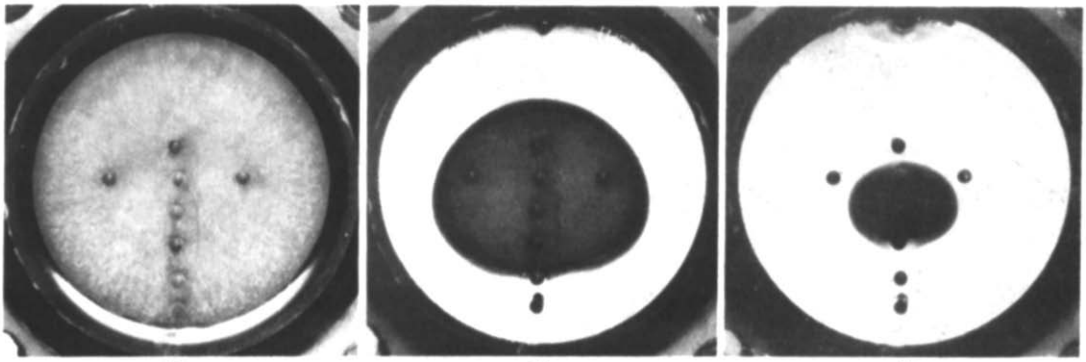


FIG. 1. Schematic representation of the test apparatus.

Melting phenomena

Figure 2 shows typical results of the melting phenomena under the condition of $Ra_D = 4.68 \times 10^7$. At the dimensionless time of $\tau = 0.0083$, the solid–liquid interface and the cylinder wall show concentricity, as shown in Fig. 2(a). However, at $\tau = 0.033$ in Fig. 2(b), it is shown that the heat transfer by natural convection in the liquid significantly affects the melting process. That is, the liquid adjacent to the cylinder wall moves upward resulting from smaller density due to heat from the wall; at the top region of the cylinder, the liquid changes its flow direction downward; thus, the heat from the wall is transferred to the top of the melting solid by natural convection. On the other hand at the bottom region of the cylinder, a three-dimensional flow instability is induced [1], and, therefore, the heat transfer from the cylinder is promoted also in this region. As a consequence, melting is accelerated at both the top and bottom regions of the unmelted solid and the cross-section of the solid is flat as shown in Fig. 2(b). Because the heat transfer rate in the top region of the unmelted solid is, in



(a) (b) (c)

FIG. 2. Melting phenomena represented by a sequence of photographs, $Ra_D = 4.68 \times 10^7$, $Pr = 52.2$: (a) $\tau = 0.0083$; (b) $\tau = 0.033$; (c) $\tau = 0.049$.

general, larger than that in the bottom region, the center of gravity of the solid shifts downward from the cylinder axis. Melting proceeds with the eccentricity kept as shown in Fig. 2(c). The melting phenomena described above are not significantly different from those observed by Rieger *et al.* [3] for smaller Rayleigh numbers.

The eccentricity of the unmelted solid ε was defined by

$$\varepsilon = \frac{\left(\begin{array}{c}\text{Distance between the cylinder axis and the}\\ \text{center of gravity of unmelted solid}\end{array}\right)}{(D-d)/2} \qquad (1)$$

where D is the inner diameter of the cylinder and d the equivalent diameter of the unmelted solid. The value of eccentricity obtained in this experiment was, for example, $\varepsilon = 0.21$ for $\tau = 0.049$ under the conditions shown in Fig. 2(c). With reference to the report by Rieger *et al.* [3], it can be estimated that $\varepsilon = 0.36$ for $Ra_D = 3.2 \times 10^6$ and $\tau = 0.08$, and that $\varepsilon = 0.23$ for $Ra_D = 8 \times 10^6$ and $\tau = 0.05$. Kuehn and Goldstein [5] investigated natural convective heat transfer in both concentric and eccentric horizontal cylindrical annuli and reported that the effect of eccentricity on the mean heat transfer coefficient was only 5% for $\varepsilon = 2/3$. In the light of their results, the effect of eccentricity of the unmelted solid (about $\varepsilon < 0.4$) on the melting process can be considered to be negligible.

ANALYSIS

Analytical model

If it is assumed that the effect of eccentricity on the melting process is neglected and that the unmelted solid is replaced by a circular cylinder having the same volume of the solid, the melting problem inside a cylinder will be resolved into a problem of natural convection heat transfer in an annulus between horizontal concentric cylinders as shown in Fig. 3. For the natural convection problem between horizontal concentric cylinders, the gap width L is generally used as a reference length. For the melting process inside a cylinder, however, the gap width is not uniform and is locally small, because the unmelted solid is flat due to the natural convection effect as described in Fig. 2. Therefore, the effective gap width L' for the melting process, which is smaller than the gap width L, is considered. The effective gap width L' is defined as

$$L' = \phi L \qquad (2)$$

where $\phi = 1$ represents concentric cylinders and $\phi < 1$ the melting process inside a cylinder.

For natural convection heat transfer between concentric horizontal cylinders, Kuehn and Goldstein [5–7] proposed empirical formulas applicable in the Rayleigh number range $2.2 \times 10^2 \leqslant Ra_L \leqslant 7.7 \times 10^7$, which includes regions of conduction, laminar convection, and partially turbulent convection. Their empirical formulas are given by

$$Nu_d = \frac{2}{\ln\left\{1 + \dfrac{2}{[(0.65 G_1 \overline{Ra}_d^{1/4})^{15} + (0.12 \overline{Ra}_d^{1/3})^{15}]^{1/15}}\right\}} \qquad (3a)$$

$$Nu_D = \frac{-2}{\ln\left\{1 - \dfrac{2}{[(0.587 G_2 \overline{Ra}_D^{1/4})^{15} + (0.12 \overline{Ra}_D^{1/3})^{15}]^{1/15}}\right\}} \qquad (3b)$$

$$(T_w - T_b)/(T_b - T_f) = Nu_d/Nu_D \qquad (3c)$$

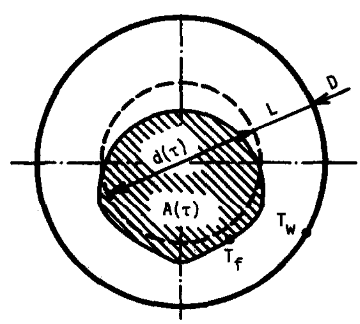


FIG. 3. Schematic representation of the analytical model.

$$Nu_{\text{conv}} = \left[\frac{1}{Nu_d} + \frac{1}{Nu_D}\right]^{-1} \tag{3d}$$

$$Nu_{\text{cond}} = 2/\ln(D/d) \tag{3e}$$

$$Nu = [Nu_{\text{cond}}^{15} + Nu_{\text{conv}}^{15}]^{1/15} \tag{3f}$$

$$k_{\text{eq}} = Nu/Nu_{\text{cond}} \tag{3g}$$

$$G_1 = \left[1 + \left(\frac{0.559}{Pr}\right)^{3/5}\right]^{-5/12} \tag{3h}$$

$$G_2 = \left[\left(1 + \frac{0.6}{Pr^{0.7}}\right)^{-5} + (0.4 + 2.6Pr^{0.7})^{-5}\right]^{-1/5} \tag{3i}$$

where $\overline{Ra}$ denotes that a bulk temperature of liquid, T_b, is used as a reference temperature, $(T_b - T_f)$. The value of k_{eq} in equation (3g) represents the ratio of transferred heat rate by natural convection to that by thermal conduction. Therefore, k_{eq} gives the equivalent thermal conductivity of natural convective liquid as follows:

$$\lambda_{\text{eq}}(t) = k_{\text{eq}}(t)\lambda_l \tag{4}$$

where λ_l is the thermal conductivity of liquid. In equation (4), the $k_{\text{eq}}(t)$ and $\lambda_{\text{eq}}(t)$ are represented as a function of time, since the heat transfer rate by natural convection varies as melting proceeds.

Equivalent thermal conductivity

If it is assumed in the melting process that (1) natural convection does not occur in the liquid, (2) the solid–liquid interface is maintained concentrically on an equivalent thermal conductivity $\lambda_{\text{eq}}(t)$ of the liquid and (3) the melting is slow enough to be applied as a quasi-steady-state assumption, the heat rate transferred from the cylinder wall to the solid–liquid interface per unit length of cylinder, $q_L(t)$, will be given by the solution for a steady-state conduction problem of a cylinder

$$q_L(t) = 2\pi\lambda_{\text{eq}}(t)\frac{T_w - T_f}{\ln[D/d(t)]} \tag{5}$$

where $d(t)$ is the equivalent diameter of unmelted solid at time t. The temperature difference $\theta(t) = T(t) - T_f$ at $r = r$ in the liquid is given by

$$\theta(t) = (T_w - T_f)\frac{\ln[2r/d(t)]}{\ln[D/d(t)]}. \tag{6}$$

Let us suppose that the solid–liquid interface at $r = s(t)$ shifts with melting by the distance $\mathrm{d}s(t)$ at time intervals of $\mathrm{d}t$, the heat balance equation at the interface is given by

$$-2\pi s(t)\rho_s L_f\,\mathrm{d}s(t) = 2\pi s(t)\lambda_{\text{eq}}(t)\left.\frac{\partial\theta(t)}{\partial r}\right|_{r=s(t)}\mathrm{d}t \tag{7}$$

where L_f is a latent heat of fusion. Substitution of equation (6) into equation (7) and integration of equation (7) with the boundary conditions $d = D \sim d(t)$ at $t = 0 \sim t$ leads to

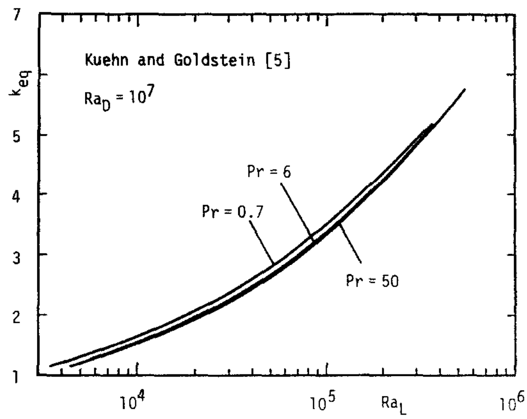


FIG. 4. Effect of Prandtl number on k_{eq}.

$$A(\tau)[1 - \ln A(\tau)] = 1 - 4\int_0^{\tau} k_{\text{eq}}(\tau)\,\mathrm{d}\tau \tag{8}$$

where it is assumed that the density of PCM does not change with melting. The $A(\tau) = d^2(\tau)/D^2$ means a dimensionless area of unmelted solid, where τ is a dimensionless time. The melting rate can, therefore, be calculated from equation (8) if the formula of $k_{\text{eq}}(\tau)$ is known. It should be noted that, for the value of $k_{\text{eq}}(\tau) = 1$, equation (8) represents the phase change process controlled by pure thermal conduction.

The values of k_{eq} for concentric annuli can be calculated from equations (3a)–(3i) given by Kuehn and Goldstein [5]. The results are shown in Fig. 4, where k_{eq} is plotted against Rayleigh numbers defined by the gap width L as a reference length. It is seen that the value of k_{eq} increases with Ra_L, since the natural convection effect becomes significant for larger L. In Fig. 4, k_{eq} is also shown for the Prandtl numbers of air, water and n-octadecane. For $Pr = 0.7$, k_{eq} takes slightly larger values than those for water and n-octadecane. However, for $Pr = 6$ and 50, it is shown that the values of k_{eq} are almost the same. The independency of k_{eq} on Prandtl number was also reported by Itoh *et al.* [8] and they arranged the Nusselt number data in the range of $5 \leqslant Pr \leqslant 800$ as a function of Rayleigh number alone.

In Fig. 5, the values of k_{eq} for $Pr = 50$ are shown as a parameter of Ra_D, where D is the diameter of the outer cylinder of concentric annuli. It is shown that k_{eq} increases with decreasing Ra_D, which means that, for the same values of L, the natural convection effect becomes significant with decreasing D. From Fig. 5, it is realized that k_{eq} is a function of Ra_L as well as Ra_D. Itoh *et al.* [8] arranged k_{eq} as a function of Rayleigh number defined by $(dD)^{0.5}\ln(D/d)$ as a reference length. However, this reference length cannot be used for the melting process in the present study, because the value of $(dD)^{0.5}\ln(D/d)$ becomes zero as melting proceeds $(d \to 0)$. In the present study, the following formula is, therefore, proposed:

$$k_{\text{eq}} = 0.228Ra_L^{1/4}\left(1 - \frac{d}{D}\right)^{1/4} \tag{9}$$

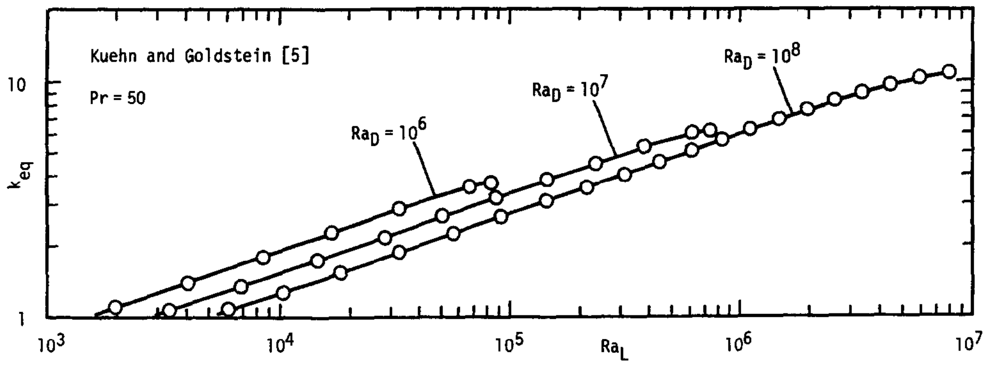


FIG. 5. Effect of Rayleigh number on k_{eq}.

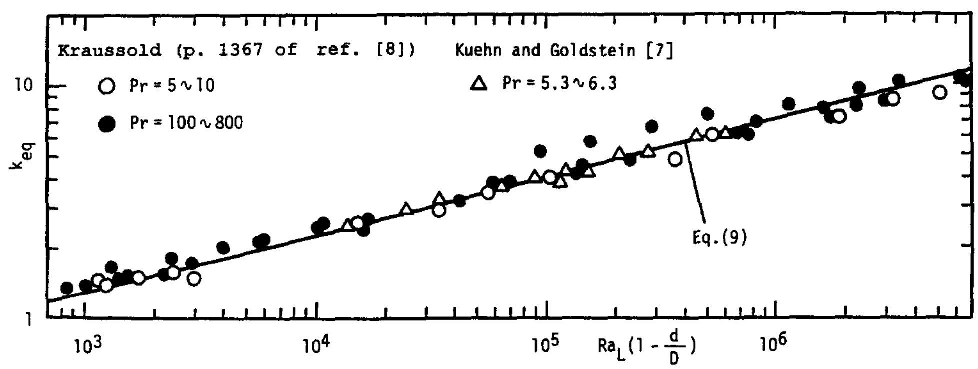


FIG. 6. Correlation equation for k_{eq}.

where the applicable range is for $1 \leqslant k_{eq} \leqslant 15$ and $k_{eq} = 1$ for $k_{eq} < 1$. Equation (9) coincides with the empirical formula obtained by Kuehn and Goldstein [7] for $D/d = 2.6$ with water, $k_{eq} = 0.202 Ra_L^{0.25}$.

In Fig. 6, the values of equation (9) are compared with the experimental data obtained by Kuehn and Goldstein [7] and by Kraussold.† The comparison is made for a wide range of Prandtl numbers, since the effect of Pr on k_{eq} is known to be small from the results of Fig. 4. In the report by Itoh *et al.* [8], it was observed that Kraussold's data reduced by using $(dD)^{0.5}\ln(D/d)$ and deviated slightly from the empirical equation at small Rayleigh numbers. In Fig. 6, however, it is shown that a good correlation is obtained by using equation (9) for a wide range of Rayleigh numbers.

Equation (9), which is obtained from the results for concentric cylinder annuli, can now be applied for the melting process. Taking into account that the effective gap width is given by equation (2) and that the natural convection effect is a function of time, we can transform equation (9) as follows:

$$k_{eq}(\tau) = 0.228\phi Ra_L(\tau)^{1/4}\left[1-\frac{d(\tau)}{D}\right]^{1/4} \quad (10)$$

where $d(\tau)$ is an equivalent diameter of unmelted solid at time τ and $Ra_L(\tau)$ is a Rayleigh number defined by a reference length of $[D-d(\tau)]/2$. The dimensionless melting rate $1-A(\tau)$ can now be estimated by substituting equation (10) into equation (8). In the following calculations, the time step of the numerical integration in equation (8) was given as $\Delta\tau = 0.001$, since it was checked that the calculation error was less than 4% for $\Delta\tau < 0.001$ in the range of $Ra_D \leqslant 10^8$.

RESULTS AND DISCUSSIONS

In Fig. 7, the analytical results of the dimensionless melting rate are compared with the experimental data, where the value of ϕ is taken as 0.9. The dashed lines in Fig. 7 indicate the phase change process controlled by thermal conduction. It is shown that the melting proceeds faster than the case of pure conduction due to the natural convection effect in the liquid phase. Figure 7(a) shows this effect for comparatively large Rayleigh numbers. Kuehn and Goldstein [5] reported in their study on eccentric cylinder annuli that the natural convection flow begins to oscillate at $Ra_L = 2\times10^5$ and becomes turbulent in the top region of the annuli at $Ra_L = 1.6\times10^7$. Although the Rayleigh number range in Fig. 7(a) is considered to be in a turbulent flow region, the analytical results coincide with the experimental data. A close observation of Fig. 7(a) reveals that the analytical results give slightly smaller values than the data just before the end of the

† See p. 1367 of ref. [8].

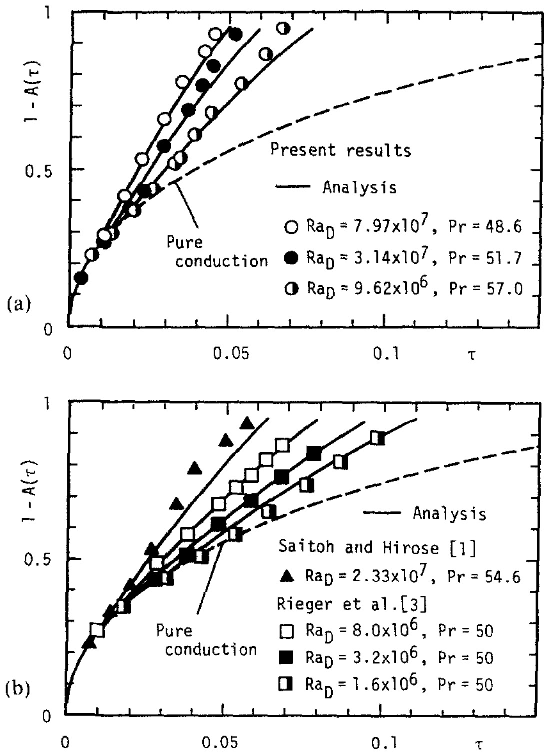


FIG. 7. Comparison between the analytical and experimental results, $\phi = 0.9$: (a) comparison for larger Rayleigh numbers; (b) comparison with previous data.

melting process. It is considered that the effects of both eccentricity and flatness of the unmelted solid on natural convection become significant as melting proceeds. This implies that ϕ in equation (2) should be defined as a function of time. Figure 7(b) shows this for comparatively smaller Rayleigh numbers and good agreement is also obtained. It is noted that the present analytical results agree with the experimental data by taking $\phi = 0.9$.

According to the numerical results by Saitoh and Hirose [1] and by Rieger *et al.* [3] for the melting process inside a cylinder, the parameters such as Stefan number, cylinder diameter and thermal diffusivity of PCM are also involved. Sasaguchi *et al.* [9] examined the melting process of a latent heat storage unit with a finned tube and noted that the effect of Stefan number on melting rate was negligibly small. However, the effects of cylinder diameter as well as thermal diffusivity of PCM have not been fully investigated so far. The physical meaning of ϕ, which is defined by equation (2), is the ratio of motive force by natural convection for the melting process to that for the eccentric cylinders. It is, therefore, induced that the value of $\phi = 0.9$ has a generality if the melting phenomena as shown in Fig. 2 do not change largely with cylinder diameter and thermal diffusivity of PCM. As far as the PCMs which are utilized for the latent heat storage system are concerned, the values of those thermal properties are nearly close to those

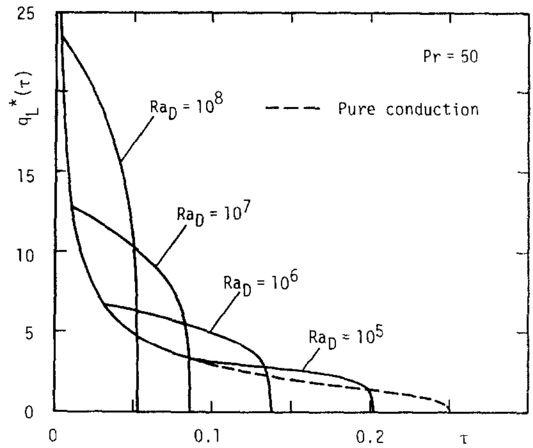


FIG. 8. Variation of $q_L^*(\tau)$ for various Ra_D.

of n-octadecane and, therefore, the melting phenomena are expected to be similar to those shown in Fig. 2.

In Fig. 8, the dimensionless values of the transferred heat rate from the cylinder to the solid–liquid interface are shown, where the sensible heat of the liquid was neglected compared to the latent heat. Substituting equation (4) into equation (5), we have

$$q_L^*(\tau) = -\frac{4}{\ln A(\tau)} k_{eq}(\tau) \tag{11}$$

where $q_L^*(\tau)$ is $q_L(\tau)/[\pi\lambda_l(T_w - T_f)]$ and $k_{eq}(\tau)$ is calculated from equation (10) for $\phi = 0.9$. In Fig. 8 it is shown that the melting proceeds faster than that of pure heat conduction due to the increase in the rate of transferred heat at the early stage of melting. It is also shown that the effect of natural convection on $q_L^*(\tau)$ becomes smaller with decreasing Ra_D and can be negligible for $Ra_D < 10^5$. This result gives almost the same value as that reported by Saito *et al.* [10] obtained using naphthalene as a PCM, where the natural convection effect is concluded to be negligible for $Gr < 3 \times 10^3$.

In the present study, n-octadecane is chosen as a PCM; however, water is also used as a PCM for a low temperature thermal energy storage system. It is useful to discuss the applicability of the present analytical method to the melting process of ice inside a horizontal cylinder. Rieger and Beer [11] examined the melting phenomena of ice inside an isothermally heated horizontal cylinder for wall temperatures in the range $4°C \leqslant T_w \leqslant 15°C$. It was observed that the shapes as well as the eccentricity of the melting ice body are quite different from those of n-octadecane at T_w = 6, 8.5 and 10°C due to the density inversion of water at 4°C. However, for $T_w > 10°C$, it was obtained that the melting phenomena are very similar to those of Fig. 2. It is, therefore, deduced that the present analytical method is applicable to the melting process of ice for $T_w > 10°C$, using the heat transfer coefficient between horizontal concentric cylinders with density inversion of water.

CONCLUSIONS

The analysis for the melting process was made by using an equivalent thermal conductivity for the liquid phase. The natural convection heat transfer of the melting process was estimated from the empirical formulas for concentric cylinder annuli. Experiments for the melting process inside an isothermally heated horizontal cylinder were also performed using n-octadecane as a PCM. The following conclusions may be drawn.

(1) The equivalent thermal conductivity for concentric cylinder annuli can be well correlated by equation (9) for $5 \leqslant Pr \leqslant 800$, where the Rayleigh number is defined by $[D-d(\tau)]/2$ rather than by $(dD)^{0.5} \ln (D/d)$ as a reference length.

(2) An analytical method for the melting process inside an isothermally heated horizontal cylinder was proposed and the melting rate of PCM can be estimated from equations (8) and (10).

(3) The effect of natural convection on the transferred heat rate from the cylinder to the solid–liquid interface is represented by equation (11) and becomes negligibly small for $Ra_D < 10^5$.

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ANALYSE DU TRANSFERT THERMIQUE AVEC CONDUCTIVITE THERMIQUE EQUIVALENTE DE LA PHASE LIQUIDE PENDANT LA FUSION DANS UN CYLINDRE HORIZONTAL CHAUD ISOTHERME

Résumé—Une méthode d'analyse avec conductivité thermique equivalente de la phase liquide est présentée pour le mecanisme de fusion d'un matériau à changement de phase (PCM) dans un cylindre horizontal chaud isotherme. Cette conductivité, pendant la fusion, est estimée à partir des résultats de la convection naturelle entre des cylindres concentriques horizontaux. La fusion est analysée ainsi comme un problème simple de conduction thermique. Des expériences sont conduites avec du n-octadecane comme PCM. Les résultats analytiques montrent pour les vitesses de fusion un bon accord avec l'expérience, pour un large domaine du nombre de Rayleigh. On montre aussi que l'effet de la convection naturelle sur le mécanisme de fusion peut être négligé pour $Ra_D < 10^5$.

UNTERSUCHUNG DES WÄRMEÜBERGANGS BEIM SCHMELZVORGANG IN EINEM ISOTHERMEN, BEHEIZTEN, HORIZONTALEN ZYLINDER UNTER VERWENDUNG EINER ERSATZWÄRMELEITFÄHIGKEIT FÜR DIE FLÜSSIGE PHASE

Zusammenfassung—Es wird eine Methode zur Darstellung des Schmelzvorgangs in Phasenwechselmaterialien (PCM) mit Hilfe der Definition einer Ersatzwärmeleitfähigkeit vorgestellt. Die Ersatzleitfähigkeit der flüssigen Phase während des Schmelzens wird mit Hilfe bekannter Aussagen über natürliche Konvektion zwischen konzentrischen horizontalen Kreiszylindern abgeschätzt. Der Schmelzvorgang wird dann als reines Wärmeleitproblem betrachtet. Begleitende Experimente werden mit n-Oktadekan als PCM durchgeführt. Die Übereinstimmung mit dem analytischen Ergebnis ist für einen weiten Bereich von Rayleigh-Zahlen gut. Es kann außerdem gezeigt werden, daß die Einflüsse der natürlichen Konvektion auf den Schmelzvorgang für $Ra_D < 10^5$ vernachlässigt werden können.

АНАЛИЗ ТЕПЛОПЕРЕНОСА НА ОСНОВЕ ЭКВИВАЛЕНТНОЙ ТЕПЛОПРОВОДНОСТИ ЖИДКОЙ ФАЗЫ В ПРОЦЕССЕ ПЛАВЛЕНИЯ ВНУТРИ ИЗОТЕРМИЧЕСКИ НАГРЕВАЕМОГО ГОРИЗОНТАЛЬНОГО ЦИЛИНДРА

Аннотация—С использованием понятия эквивалентной теплопроводности жидкой фазы проведен анализ теплопереноса при плавлении вещества в изотермически нагреваемом горизонтальном цилиндре. Коэффициент эквивалентной теплопроводности жидкой фазы в процессе плавления определяется по результатам естественноконвективного теплопереноса между концентрическими горизонтальными цилиндрами круглого сечения. Таким образом, плавление анализируется только как задача теплопроводности. Описываются также эксперименты с н-октадеканом, используемым в качестве вещества, претерпевающего фазовые изменения. Полученные аналитические результаты для скоростей плавления показывают хорошее соответствие с экспериментальными данными в широком диапазоне значений числа Рэлея. Показано также, что при $Ra_D < 10^5$ влияние естественной конвекции на процесс плавления пренебрежимо мало.